Analyzing and Addressing Common Mathematical Errors in Secondary Education

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Abstract

After much research and discussion with mathematics educators, it is apparent that secondary education students make many common errors in mathematics. As a result, it is important to analyze what causes and how to address these mathematical errors so that teachers can correct and prevent these errors within the classroom. Therefore, I have briefly researched and analyzed the causes of mathematical errors in secondary education and ways to address students' errors. Then, through research, classroom observations, and discussions with secondary education teachers, I compiled a list of common student errors in mathematics and created activity sheets that address each error individually. To coincide with my research findings, the activity sheets confront each error conceptually and incorporate various mathematical representations.

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I would also like to thank Mrs. Brumley, Miss Lassiter, and Mr. Gill for allowing me into their classrooms and for contributing to my list of common mathematical errors among secondary education students.

Through research and discussion with mathematics educators, it is evident that students make common errors in mathematics. Therefore, before identifying the specific errors, it is crucial to discuss why students make these errors in general. It is important to note that the origin of mathematical errors have many similarities and differences.

Students' Errors

Calculation Errors

It is true that some student errors in mathematics may simply be the result of carelessness and a short attention span. Carelessness and lack of attention can result in calculation errors. Within the article "Errors that are Common in Multiplication," the authors categorize calculation errors as mistakes in the addition and multiplication of numbers (Killian, Cahill, Ryan, Sutherland, & Taccetta, 1980). However, since the study in this article mainly focused on multiplication, it may be more beneficial to generalize calculation errors as mistakes in the addition, subtraction, multiplication, and division of numbers. For example, a student may carelessly recall a basic addition fact incorrectly while solving a mathematical problem. As a result, the student's process or procedure may be correct, but his or her final answer would be incorrect.

Procedural Errors

However, research indicates that many students also make mathematical errors not only because of carelessness but also because they do not have a conceptual understanding of mathematics. For example, in a study discussed in the article "Errors that are Common in Multiplication" it was determined that forty-one percent of the errors in the study discussed in the article were calculation errors, while fifty-six percent of the errors were procedural errors. "Procedural errors were defined as errors that involved a violation of the algorithmic procedure such as misplacing digits or performing operations in the wrong order" (Killian et al., 1980, p. 23). These types of errors suggest that students do not understand the concept related to the procedure. As a result, students do not have an understanding of why or how a procedure works; therefore, students do not recognize the importance applying and computing the procedure correctly. For example, as discussed in a portion of the article "Some Cognitive Factors as Causes of Mistakes in the Addition of Fractions," a student recalls that addition and multiplication are both used in the algorithm for adding fractions, but the student uses addition and multiplication in the wrong order. This error would be classified as a procedural error; however, more specifically, the authors refer to this type of error as a wrong reconstruction of details (Vinner, Hershkowitz, & Bruckheimer, 1981). Without a strong conceptual understanding of fractions, the student has difficulty appropriately reconstructing the algorithm for adding fractions. If the student had a conceptual understanding of how a common denominator can help add fractions by putting the fractions into the same units, the student would understand why he or she would use multiplication to find a common denominator before using addition when applying the algorithm for adding fractions. "Hence, if the concept does not exist and there is only its formal representation, mistakes are very likely to occur" (Vinner et al., 1981, p. 72). Therefore, it is pertinent that students develop a conceptual understanding in addition to a procedural understanding when learning mathematics.

Classifying Errors

In addition to classifying students' mathematical errors as simply calculation or procedural errors, it may also be helpful to look at other classifications that researchers have identified for categorizing mathematical errors. For example, in the article "Some Cognitive Factors as Causes of Mistakes in the Addition of Fractions" the authors discuss how errors were categorized in a sample of students who were questioned about the addition of fractions (Vinner et al., 1981). These categories could very easily be generalized for most mathematical errors, not just the addition of fractions.

Procedural Errors

Misidentification. One category was misidentification. Misidentification occurs when a student falsely applies an algorithm that worked successfully on one problem to a problem in which the algorithm is not appropriate (Vinner et. al, 1981). Students recognize similarities among problems and rather than further analyzing the problems to find differences, the students apply a procedure that worked for a problem that had similar characteristics. In the book *Error Patterns in Computation*, author Robert B. Ashlock (2006) contributes these types of errors to overgeneralization. "Many misconceptions and erroneous procedures are generated as students overgeneralize during the learning process" (Ashlock, 2006, p. 18). For example, a student may erroneously overgeneralize the Pythagorean Theorem and apply it to non-right triangles. If the student does not have a conceptual understanding of why the Pythagorean Theorem applies to right triangles, it may be easy for students to overgeneralize and to apply the Pythagorean Theorem to any problem in which a triangle is involved or present.

Misgeneralization. This type of error also corresponds to the misgeneralization theory and the repair theory that are discussed in the article "Analyzing and Modeling Arithmetic Errors" (Blando, Kelly, Schneider, & Sleeman, 1989). "The misgeneralization theory suggests that some errors result when a student infers 'several rules which are consistent with the example, and not just the correct rule' " (Blando et al., 1989, p. 301) For example, the authors state that a student who has never added together a two and a three digit number may believe that addition is always between two numbers. As a result, since one of the numbers is lacking a third digit, the student misgeneralizes that numbers in the left-most column are added rather then just numbers in the same column (Blando et al., 1989).

Repair theory. On the other hand, the repair theory states that errors occur when students do not understand a particular aspect of a problem or the students encounter a difficult problem. Then, students erroneously modify a procedure they are comfortable with and incorrectly apply the modified procedure to the problem. "These errors occur so frequently that educators can predict the type of mistakes that are likely to be made, which explains why students tend to make the same errors throughout their schooling" (Oppenheimer & Hunting, 1999, p. 318). The repair theory occurs because it is human tendency to focus on familiar parts or least difficult parts of a problem.

Overspecialization. Lastly, in *Error Patterns in Computation*, Ashlock (2006) refers to "overspecializing." Overspecializing is the opposite of overgeneralization. When students overspecialize, the students mistakenly restrict procedures. Ashlock (2006) states "For example, a student may decide that in order to add or subtract decimals, there must be the same number of digits on either side of the decimal point" (p. 20). Obviously, such as restriction would severely limit the decimals that the student may believe he or she can subtract.

Symbolic Errors

In addition to misconceptions about procedures and rules, mathematical symbols also lead to errors. According to authors Oppenheimer and Hunting (1999), "As students progress through school, teachers begin to use symbols to represent mathematical ideas, and the symbols begin to take on a life of their own" (p. 318). As a result, students begin to make errors by falsely relating mathematical problems that use similar symbols. Students try to create meaning in the patterns of mathematical symbols and signs that they see in front of them rather than trying to understand what they are actually doing. This search for patterns in the symbols leads to misinterpretations that in turn result in mathematical errors. "Confusion arises when the symbolic configuration of a problem is similar to problems learned earlier, and students end up using inappropriate rules" (Oppenheimer & Hunting, 1999, p. 318). Research shows that those students who improve and make fewer errors do not do so by learning conceptual, but they become better at deciphering differences in notation (Oppenheimer & Hunting, 1999). Therefore, it is apparent that understanding and recognizing differences in mathematical notation is also crucial in preventing mathematical errors among students.

Wrong interpretation of symbols. This leads to another category, the wrong interpretation of symbols, as listed in the article "Some Cognitive Factors as Causes of Mistakes in the Addition of Fractions" (Vinner et al., 1981). For example, students may also improperly misinterpret the plus sign to mean they need to add both the numerator and the denominator of fractions. This type of error would suggest that the students do not have a conceptual understanding of fractions but also that students do not interrupt the plus sign as representing the addition of the two entire quantities.

Improper use of symbols. Also, many errors are made due to the improper use of symbols, such as inequality signs and radicals. In particular, students have difficulty recognizing the difference between symbols that represent something, such as an inequality sign representing greater than or less than, as compared to signs that represent operators and signal the student to perform a mathematical operation. This difficulty in interrupting mathematical symbols and notation contributes to mathematical errors.

From the information presented, it is apparent that notation, language, preconceptions, and the lack of conceptual understanding all play significant roles in mathematical errors.

However, now it is important to consider how teachers can use this information to appropriately address and prevent mathematical errors. An obvious key is teaching conceptual understanding before procedural understanding to ensure that students have a deeper understanding of the meaning behind the mathematical symbols, notation, and language that they use.

Preventing and Correcting Students Mathematical Errors and Misconceptions

Addressing Calculation Errors and Careless Mistakes

First, it might be necessary to address careless mathematical mistakes. These are mainly calculation errors and errors that occur possibly due to carelessness or lack of an attention span. A possible solution is the incorporation of an error checklist into the regular classroom routines and procedures. In the article "Careless Mistakes," Karen McAcy (1983) states, "I provide a checklist of the most commonly made mistakes, such as 'added wrong,' 'dropped the negative,' 'did not distribute the negative,' 'copied wrong,' and so on. The list includes only mistakes resulting from inattention to details; failure to understand a topic is not carelessness" (p. 299). If each student had a checklist and marked the type of mistakes they made after reviewing graded assignments, the students can note their most common mistakes. With this awareness, students may be more careful and address these mistakes on their next homework assignments. The checklist serves as a resource for students to self-assess their assignments in mathematics.

Addressing Procedural Errors

Introducing the concept before the procedure. However, since many mathematical errors are not the result of carelessness, it is important to address how to prevent and correct other mathematical errors that are common among students. "Conceptual learning in mathematics always focuses on ideas and on generalizations that make connections among ideas. In contrast, procedural learning focuses on skills and step-by-step procedures" (Ashlock, 2006, p. 7). Behind every mathematical procedure is a mathematical concept or a series of mathematical concepts. Without understanding the concepts, students will simply plug-and-chug without applying meaning and understanding to mathematical procedures. Without meaning and understanding, students make errors without recognition. Therefore, the first step in preventing and correcting mathematical errors should be to teach the mathematical concept before the mathematical procedure is introduced.

Building networks. The next consideration is how to teach the material conceptually. In "Learning and Teaching with Understanding" authors Hiebert and Carpenter (1992) suggest creating a network of representations and relationships. The networks are created through connections. These connections may be similarities or differences between various representations or simply connecting a representation to a mathematical procedure. Then, as students build their knowledge, they begin to reorganize these networks in order to form new connections and modify old connections. In general, networks will help students learn how to use their conceptual knowledge when working mathematical problems.

As previously stated, many mathematical errors are the result of misconceptions, therefore, creating networks will help students recognize where they have made legitimate or inappropriate connections. "Unless students are forced to confront explicitly the conflict between their misconceptions and the scientific principles they have learned, the connections may never be made; the misconceptions and the scientific principles may coexist as separate islands of knowledge" (Hiebert & Carpenter, 1992, p. 89). Therefore, it is important to take a more in-depth look into how to develop networks.

Concrete manipulatives. Networks can be connections between various representations; therefore, one possible representation format is concrete representations. Research indicates that students can learn from concrete manipulatives, such as base-ten blocks. Such physical objects are beneficial because they help students make connections between concrete mental representations and symbolic mental representations. However, the manipulatives are only beneficial if presented correctly. The context in which the manipulatives are being used must be considered. In addition, depending on the mathematical topic being represented concretely, prior knowledge or pre-networks may already need to be in place for the manipulations of the materials to be beneficial (Hiebert & Carpenter, 1992).

Real-world applications. In addition to concrete representations, it is important that the representations are meaningful and the students should be able to relate to the representations. "Research supports the belief that students are better able to develop meaning for symbols and procedures when they can draw on referents that are meaningful to them" (Oppenheimer & Hunting, 1999, p. 319). Therefore, it is important to take into consideration the cultural environments that surround students and to choose and develop representations that pertain to these environments. As a result, in addition to connecting procedural learning to conceptual learning it is also important that both are connected to real-life applications.

Variety of formats and representations. Next, when rules and procedures are presented, they should be presented in several different formats. "Teachers should teach a mathematical rule by using many different formats, so that students can more readily learn that a rule abstracted from one format may not apply in another" (Blando et al., p. 306). It is important that students recognize similarities and make connections, but sometimes students recognize similarities and make connections that when shown a different format no longer seem to be appropriate or hold true. According to those who study thinking, one of the greatest errors of inductive learning is the failure to consider enough examples before drawing a conclusion.

(Ashlock, 2006). "They look at commonalities among their initial contacts with an idea or procedure. They form an abstraction with certain common characteristics, and their concept or algorithm is formed" (Ashlock, 2006, p. 17). Including several examples of different formats when teaching a new concept helps students to decipher and eliminate any erroneous preconceptions.

Reflection and self-assessment. In addition to using various formats and representations, teachers should also encourage students to reflect on their mathematical procedures. In the article, "Making the Most of Student Errors," the authors state, "To recognize and successfully deal with their errors, students must understand how to recognize and successfully reflect on their errors" (Lannin, Arbaugh, Barker, & Townsend, 2006, p.182). In most cases, students do not recognize their error and if they do have doubts about their mathematical procedures, they often to not possess the conceptual knowledge needed to check their solutions (Vinner et al., 1981, p. 73). Therefore, self-assessment is also a critical part of correcting and preventing mathematical errors.

The authors of "Making the Most of Students Errors" discuss four ways to help students self assess their errors. First, the classroom environment must create opportunities for students to discover and recognize their own errors. This can be accomplished through student interaction and discussion. A productive strategy may be to have students think aloud. This not only requires students to think about their thinking, but other students can compare their thoughts to other students' thinking. In the article "Why Students Make Math Errors," authors VanDevender and Harris (1987) state, "Encouraging the student to 'tell his thinking' is a useful technique to help the student become aware of his thought processes. This further provides students with an effective strategy for learning to assess their own strengths and weaknesses in math" (p. 81).

Next, students should be encouraged to examine their errors. Rather than simply focusing on right and wrong, students should focus on finding the procedures or strategies that caused their errors. Thirdly, it is important to help students recognize the source of their error. This can be accomplished through questioning and metacognition. "Research has demonstrated that children can be taught these strategies, including the ability to predict outcomes, explain to oneself in order to improve understanding, note failures to comprehend, activate background knowledge, plan ahead, and apportion time and memory" (National Research Council, 2000, p. 18).

Lastly, encourage students to consider how they know when they have found and correctly fixed an error (Lannin, et. al, 2006). This can be accomplished through the use of diagrams or written responses. In general, the students must learn how to reflect upon their own work and explain the mathematical reasoning behind their work.

Time. In addition to teaching conceptual learning, creating networks, and encouraging self-assessment, time is also a factor that a teacher should consider in order to eliminate mathematical errors. "Errors usually are made consistently, not because the students aren't thinking, but because they quickly find the key word they looking for. Students who make the most correct responses (and higher ability students) often need more time, because they are reflecting more" (Smith, 2002, p. 133). As a result, rather than giving students a series of problems that must be completed within a given time range, it may be more beneficial to assign fewer problems without a time constraint and combine the problems with questions that force the student to reflect upon their mathematical reasoning and the mathematical procedures they performed.

Formatting Activities that Help Correct and Prevent Students' Mathematical Errors

It is evident that there are several factors that must be considered, such as conceptual learning, representation, and time when helping to correct and prevent mathematical errors within a middle school or high school classroom. It may be helpful to consider three factors noted in "How People Learn: Brain, Mind, Experience, and School" when developing activities that address errors in a mathematics classroom.

Preconceptions

The first factor relates to students' preconceptions. A teacher must first identify the error and then note any preconceptions the student may have related to the mathematical concepts or procedures involved in the error. Students develop preconceptions about mathematical concepts before they develop a true conceptual understanding of the concepts. "If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught, or they may learn them for purposes of a test but revert to their preconceptions outside of the classroom" (National Research Council, 2000, p. 14). If students are not forced to engage their understandings, their false preconceptions may not be identified. As a result, students may not get beyond their erroneous preconceptions, and they will have difficulty learning new mathematical concepts. Therefore, the first activity when addressing a common mathematical error should engage the students' pre-existing conceptual understanding to help the students recognize any erroneous preconceptions.

Competence in Mathematics

The second factor in "How People Learn: Brain, Mind, Experience, and School" corresponds to how competent a student is in mathematics. The authors note three factors that indicate competence in an area of mathematics. The first area is developing a deep foundation of factual knowledge. The second area is understanding how the ideas and facts relate to the concept. Then, the third area is the ability to organize the knowledge for future retrieval and application. Therefore, in the initial activities, the students should be able to state facts, patterns, and connections they recognize. Then, the students should be asked to these relate facts, patterns, and connections to the mathematical concepts. Lastly, in the initial activities, the students should be asked to write, illustrate, or state their understandings so that they begin to create their own networks of connections and understanding. These networks will help the students easily retrieve the information for future use.

Metacognition

Thirdly, the last factor mentioned in "How People Learn: Brain, Mind, Experience, and School," is a metacognitive approach to instruction. Throughout the completion of each activity, the students should reflect upon their understandings. The students must be able to recognize inappropriate connections they have made. In addition, students must be able to relate any new knowledge to their pre-existing knowledge. Lastly, students must reflect upon their organization of the mathematical concepts and facts. Students must be able to recall and apply this information in the future if they want to be successful and competent in a specific area of mathematics. Therefore, throughout each activity that addresses a mathematical error, the students should be asked questions that require them to explain their processes, explain their understandings, and challenge them to think critically. Also, an overall self-assessment activity that addresses a mathematical error would be beneficial. In general, the student needs to be able to evaluate their own understandings.

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Addressing Common Mathematical Errors in Secondary Education

Error Descriptions

Activity Descriptions

Activity Sheets

Error Description: Division by Zero

Error:

Students believe that any number divided by zero equals zero.

Incorrect Example: $\frac{4}{0} = 0$

Reason for Error:

Students are aware of the fact the zero divided by anything equals zero. Students then misgeneralize that anything divided by zero equals zero.

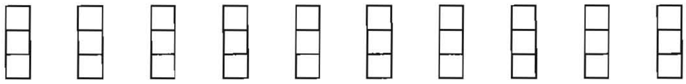
Activity #1 Description:

In the first activity, the students will illustrate the given story problems with base ten blocks. For each problem, the students will be given a total number of pieces candy and the number of pieces distributed to each person. Then, the students will be asked how many people will receive candy. Next, the students must write an equation involving division that represents the story problem they modeled with the base ten blocks. Lastly, the students will be asked to make a conclusion about $\frac{x}{0}$ based on the conclusions they make about the story problems in which zero pieces of candy are distributed to each person. The purpose of this activity is for students to realize that it is not possible to define the number of groups of zero within a given total.

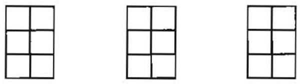
Solution:

1.) $\frac{30}{6} = 5$

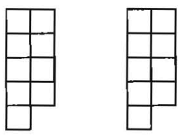
2.) $\frac{30}{3} = 10$



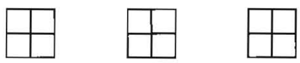
4.) $\frac{18}{6} = 3$



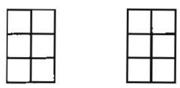
5.) $\frac{18}{9} = 2$



7.) $\frac{12}{4} = 3$



8.) $\frac{12}{6} = 2$

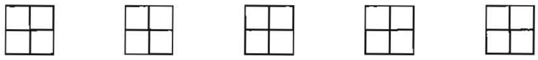


<u>Activity #2 Description:</u>

In the second activity, students will use base ten blocks to model story problems in which candy is divided evenly among a specified number of people. Then, the students will be asked to write equations involving division that illustrate the number of pieces of candy each person will receive. Next, the students will be asked if it is possible to share candy among zero people. The purpose of this activity is for students to realize that it is not possible to divide a given value by zero groups.

Solution:

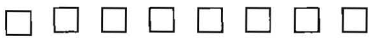
1.) $\frac{20}{5} = 4$



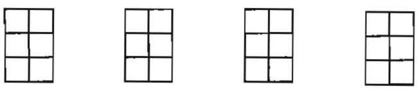
2.) $\frac{20}{2} = 10$

4.) $\frac{8}{4} = 2$

5.) $\frac{8}{8} = 1$



7.) $\frac{24}{4} = 6$



8.) $\frac{24}{3} = 8$

Activity #3 Description:

In the last activity, students will be asked to convert equations involving division into equations involving multiplication. One of the equations that the students will convert has a divisor that is zero. Once the students have converted this equation, the students will be asked if there is a value that when multiplied by zero produces a product other than zero. The purpose of this activity is for students to realize that there is no number that when multiplied by zero produces a product is that not zero; therefore, one cannot divide my zero. Lastly, to assess the students' overall understandings, the students will be asked to make a conclusion about dividing by zero based on these activities.

Solution:

1.) $\frac{45}{9} = 5$ $\quad$ $45 = 9 \times 5$

2.) $\frac{45}{0} = x$ $\quad$ $45 = 0 \times x$

1.) $\frac{32}{8} = 4$ $\quad$ $32 = 8 \times 4$

2.) $\frac{32}{0} = x$ $\quad$ $32 = 0 \times x$

1.) $\frac{27}{9} = 3$ $\quad$ $27 = 9 \times 3$

2.) $\frac{27}{0} = x$ $\quad$ $27 = 0 \times x$

Activity Sheet

Name: ______________________________

Activity #1:

1.) Suppose you have 30 pieces of candy. You plan to give each one of your friends 6 pieces of candy. How many friends will get 6 pieces of candy?

Use base ten blocks to model this scenario. Draw a sketch of your base ten blocks in the space below.

Think About It!

- How did you model this problem with the blocks?______________________________

____

- How many groups of 6 pieces of candy are in the total 30 pieces of candy?______________
- Write a symbolic equation that represents this scenario.______________________________

2.) Suppose you have 30 pieces of candy. You plan to give each one of your friends 3 pieces of candy. How many friends will get 3 pieces of candy?

Use base ten blocks to model this scenario. Draw a sketch of your base ten blocks in the space below.

Think About It!

- How did you model this problem with the blocks?______________________________

____

- How many groups of 3 pieces of candy are in the total 30 pieces of candy?______________
- Write a symbolic equation that represents this scenario.______________________________

3.) Suppose that you have 30 pieces of candy. You plan to give each one of your friends zero pieces of candy. How many friends will get zero pieces of candy?

Can you use the base ten blocks to model this scenario? Why or why not?

Think About It!

- Can you determine how many friends will get zero pieces of candy? Why or why not?____
- Write a symbolic expression that represents this scenario. Can you evaluate this expression? Why or why not?____

____

4.) Now, suppose that you have 18 pieces of candy. You plan to give each one of your friends 6 pieces of candy. How many friends will get 6 pieces of candy?

Use base ten blocks to model this scenario. Draw a sketch of your base ten blocks in the space below.

Think About It!

- How did you model this problem with the blocks?____

____

- How many groups of 6 pieces of candy are in the total 18 pieces of candy?________________
- Write a symbolic equation that represents this scenario.__________________________________

5.) Now, suppose that you have 18 pieces of candy. You plan to give each one of your friends 9 pieces of candy. How many friends will get 9 pieces of candy?

Use base ten blocks to model this scenario. Draw a sketch of your base ten blocks in the space below.

Think About It!

- How did you model this problem with the blocks?____
- How many groups of 9 pieces of candy are in the total 18 pieces of candy?__________
- Write a symbolic equation that represents this scenario.__________________________

6.) Now, suppose that you have 18 pieces of candy. You plan to give each one of your friends zero pieces of candy. How many friends will get zero pieces of candy?

Can you use the base ten blocks to model this scenario? Why or why not?

Think About It!

- Can you determine how many friends will get zero pieces of candy? Why or why not?____
- Write a symbolic expression that represents this scenario. Can you evaluate this expression? Why or why not?____

7.) Now, suppose that you have 12 pieces of candy. You plan to give each one of your friends 4 pieces of candy. How many friends will get 4 pieces of candy?

Use base ten blocks to model this scenario. Draw a sketch of your base ten blocks in the space below.

Think About It!

- How did you model this problem with the blocks?____
- How many groups of 4 pieces of candy are in the total 12 pieces of candy?__________
- Write a symbolic equation that represents this scenario.__________________________

8.) Now, suppose that you have 12 pieces of candy. You plan to give each one of your friends 6 pieces of candy. How many friends will get 6 pieces of candy?

Use base ten blocks to model this scenario. Draw a sketch of your base ten blocks in the space below.

Think About It!

- How did you model this problem with the blocks?____________________________________

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- How many groups of 6 pieces of candy are in the total 12 pieces of candy?______________
- Write a symbolic equation that represents this scenario._______________________________

9.) Now, suppose that you have 12 pieces of candy. You plan to give each one of your friends zero pieces of candy. How many friends will get zero pieces of candy?

Can you use the base ten blocks to model this scenario? Why or why not?

Think About It!

- Can you determine how many friends will get zero pieces of candy? Why or why not?____
- Write a symbolic expression that represents this scenario? Can you evaluate this expression? Why or why not?____

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- Based on these problems, what can you conclude about $\frac{x}{0}$?____________________________

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Activity #2:

1.) Suppose you have 20 pieces of candy. You evenly share the 20 pieces with 5 people. How many pieces of candy does each person receive?

Use base ten blocks to model the following scenario. Draw a sketch of your base ten blocks below the problem.

Think About It!

- How did you model this problem with the base ten blocks?______________________

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- Write a symbolic equation that represents this scenario.______________________

____

2.) Suppose you have 20 pieces of candy. You evenly share the 20 pieces with 2 people. How many pieces of candy does each person receive?

Use base ten blocks to model the following scenario. Draw a sketch of your base ten blocks below the problem.

Think About It!

- How did you model this problem with the base ten blocks?______________________

____

- Write a symbolic equation that represents this scenario.______________________

____

3.) Suppose you have 20 pieces of candy. You evenly share the 20 pieces with zero people. How many pieces of candy does each person receive?

Can you use the base ten blocks to model this scenario? Why or why not?

Think About It!

- Can you share 20 pieces of candy with zero people? Why or why not?
- Write a symbolic expression that represents this scenario. Can you evaluate this expression?

Why or why not?____

____

4.) Suppose you have 8 pieces of candy. You evenly share the 8 pieces with 4 people. How many pieces of candy does each person receive?

Use base ten blocks to model the following scenario. Draw a sketch of your base ten blocks below the problem.

Think About It!

- How did you model this problem with the base ten blocks?________________________

____

- Write a symbolic equation that represents this scenario.________________________

____

5.) Suppose you have 8 pieces of candy. You evenly share the 8 pieces with 8 people. How many pieces of candy does each person receive?

Use base ten blocks to model the following scenario. Draw a sketch of your base ten blocks below the problem.

Think About It!

- How did you model this problem with the base ten blocks?________________________

____

- Write a symbolic equation that represents this scenario.________________________

____

6.) Suppose you have 8 pieces of candy. You evenly share the 8 pieces with zero people. How many pieces of candy does each person receive?

Can you use the base ten blocks to model this scenario? Why or why not?

Think About It!

- Can you share 8 pieces of candy with zero people? Why or why not?
- Write a symbolic expression that represents this scenario. Can you evaluate this expression? Why or why not?____

____

7.) Suppose you have 24 pieces of candy. You evenly share the 24 pieces with 4 people. How many pieces of candy does each person receive?

Use base ten blocks to model the following scenario. Draw a sketch of your base ten blocks below the problem.

Think About It!

- How did you model this problem with the base ten blocks?____________________________

____

- Write a symbolic equation that represents this scenario.____________________________

____

8.) Suppose you have 24 pieces of candy. You evenly share the 24 pieces with 3 people. How many pieces of candy does each person receive?

Use base ten blocks to model the following scenario. Draw a sketch of your base ten blocks below the problem.

Think About It!

- How did you model this problem with the base ten blocks?____________________

____

- Write a symbolic equation that represents this scenario.____________________

____

9.) Suppose you have 24 pieces of candy. You evenly share the 24 pieces with zero people. How many pieces of candy does each person receive?

Can you use the base ten blocks to model this scenario? Why or why not?

Think About It!

- Can you share 24 pieces of candy with zero people? Why or why not?
- Write a symbolic expression that represents this scenario. Can you evaluate this expression? Why or why not?____________________

____

- Suppose you share x pieces of candy with zero people. Write can expression to represent this scenario. Can you evaluate this expression? Why or why not?____________________

____

Activity #3:

Transform the following equations involving division into equation involving multiplication.

1.) $\frac{45}{9} = 5$ __________

2.) $\frac{45}{0} = x$ __________

Think About It!

- Is there any value for x that makes the equations in number 2.) true? Explain.____________________

____

____

1.) $\dfrac{32}{8} = 4$ ________

2.) $\dfrac{32}{0} = x$ ________

Think About It!

- Is there any value for x that makes the equations in number 2.) true? Explain.____________

____

1.) $\dfrac{27}{9} = 3$ ________

2.) $\dfrac{27}{0} = x$ ________

Think About It!

- Is there any value for x that makes the equations in number 2.) true? Explain.____________

____

- Are there any conclusion you can make about dividing by zero based on these activities?____

____

Error Description: Negative Numbers

Error:

Students receive a positive sum when adding two negative values.

Incorrect Example: $(-2)+(-5)=7$

Reason for Error:

Students believe that two negatives numbers sum to a positive number. This error may be the result of an overgeneralization and an inappropriate connection to the multiplication of negative numbers. For example, it is possible that students have learned that a negative number multiplied by a negative number produces a positive product. Then, students overgeneralize and falsely assume that the same is true when adding two negative numbers. This error suggests that students lack a conceptual understanding of both adding and multiplying negative numbers. Therefore, both the addition and the multiplication of negative numbers need to be included in the activities that address this error. Also, students may have difficulty accepting that the sum of two negative numbers is negative if they view addition as an operation that combines values to produce a greater value.

Error Description: Adding Negative Numbers

Activity #1 Description:

In the first activity, the students will form a human number line. Have each student hold an integer value between -10 and 0. Then, have the students line up in order. Next, show the students the addition problems listed on the activity sheet. Ask the students where to begin on the number line and ask the students to identify the value that needs to be added to this initial value. Next, in order to add this value to the initial value, ask the students whether they should move left or right on the number line. Lastly, ask the students where they will land on the number line once they add the identified value. The purpose of this activity is for students to realize that when they add a negative number to another negative number, they must move left on the number line which results in a sum that is negative.

Solution:

1.) $-3+(-5)=-8$

• Where should you begin on the number line?	-3
• How far should you move?	5 units
• Should you move left or right on the number line?	left
• Where do you land on the number line?	-8

2.) $-4+(-2)=-6$

• Where should you begin on the number line?	-4
• How far should you move?	2 units
• Should you move left or right on the number line?	left
• Where do you land on the number line?	-6

3.) $-2+(-1)=-3$

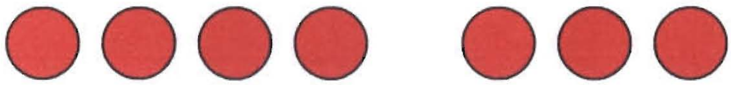
• Where should you begin on the number line?	-2
• How far should you move?	1 unit
• Should you move left or right on the number line?	left
• Where do you land on the number line?	-3
• Are any of the answers for 1.), 2.), or 3.) greater than zero?	No

Activity #2 Description:

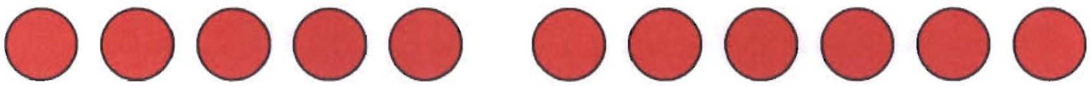
In the second activity, the students will use counting chips to add two negative numbers. The purpose of this activity is to show students that all of their chips represent a negative value and that when added together, the sum is negative.

Solution:

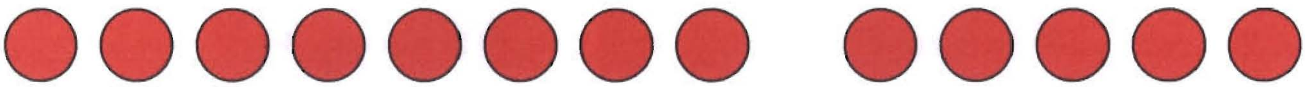
1.) $-4 + (-3) = -7$



2.) $-5 + (-6) = -11$



3.) $-8 + (-5) = -13$



- Is the sum of $(-4 + -3 + -7)$ negative or positive? Explain.
  The sum is negative. The sum is -14.

Activity Sheet

Name: ______________________________

Activity #1: **Human Number Line**

1.) $-3+(-5)=$ ______

Think About It!

- Where should you begin on the number line?______________________________
- How far should you move?______________________________
- Should you move left or right on the number line?______________________________
- Where do you land on the number line?______________________________

2.) $-4+(-2)=$ ______

Think About It!

- Where should you begin on the number line?______________________________
- How far should you move?______________________________
- Should you move left or right on the number line?______________________________
- Where do you land on the number line?______________________________

3.) $-2+(-1)=$ ______

Think About It!

- Where should you begin on the number line?______________________________
- How far should you move?______________________________
- Should you move left or right on the number line?______________________________
- Where do you land on the number line?______________________________
- Are any of the answers for 1.), 2.), or 3.) greater than zero? What conclusion can you make about the sum of two negative numbers based on this activity?______________________________

____

Activity #2:

Use counting chips to add the following. Draw a picture of the counting chips that represent your answer below.

1.) -4 + (-3)

Think About It!

- Describe how you used the chips to determine the answer.______________________________

____

2.) -5 + (-6)

Think About It!

- Describe how you used the chips to determine the answer.______________________________

____

3.) $-8+(-5)=-13$

Think About It!

- Describe how you used the chips to determine the answer.______________________________

____

- What can you conclude about the sum of two negative numbers based on this activity?______

____

- Is the sum of (-4 + -3 + -7) negative or positive? Explain.______________________________

____

Error Description: Multiplying Negative Numbers

Activity #1 Description:

In the first activity, the students will complete a few charts by finding a series of products. From one expression to the next, one of the factors in the expression is decreased by one while the other factor remains the same. Then, the students will be asked to identify how the products change as the expressions change. Lastly, the students will predict the product of the next expression in the pattern and write the next four equations in the pattern.

Solution:

1.)

$-3 \times 5 =$ -15
$-3 \times 4 =$ -12
$-3 \times 3 =$ -9
$-3 \times 2 =$ -6
$-3 \times 1 =$ -3
$-3 \times 0 =$ 0

- How does the product change as the numbers multiplied by -3 get smaller?
  The product increases by 3.
- Predict -3×-1. Explain your prediction.
  Since 3×0=0 and the values increase by 3 each time, -3×-1=3.
- Write the next four equations in the pattern.
  $-3 \times -2 = 6$ $-3 \times -3 = 9$ $-3 \times -4 = 12$ $-3 \times -5 = 15$

2.)

$-5 \times 5 =$ -25
$-5 \times 4 =$ -20
$-5 \times 3 =$ -15
$-5 \times 2 =$ -10
$-5 \times 1 =$ -5
$-5 \times 0 =$ 0

- How does the product change as the numbers multiplied by -5 get smaller?
  The product increases by 5.
- Predict -5×-1. Explain your prediction.
  Since $5\times0=0$ and the values increase by 5 each time, $-5\times-1=5$.

- Write the next four equations in the pattern.
  $-5\times-2=10$ $-5\times-3=15$ $-5\times-4=20$ $-5\times-5=25$

3.)

$-7\times5=-35$
$-7\times4=-28$
$-7\times3=-21$
$-7\times2=-14$
$-7\times1=-7$
$-7\times0=0$

- How does the product change as the numbers multiplied by -7 get smaller?
  The product increases by 7.

- Predict -7×-1. Explain your prediction.
  Since $7\times0=0$ and the values increase by 5 each time, $-7\times-1=7$.

- Write the next four equations in the pattern.
  $-7\times-2=14$ $-7\times-3=21$ $-7\times-4=28$ $-7\times-5=35$

<u>Activity #2 Description:</u>

In the second activity, the students will use a number line representation to explore why the multiplication of two negative numbers produces a positive product.

Solution:

1.) $-5\times-6=30$
2.) $-4\times-7=28$
3.) $-2\times-9=18$

Activity Sheet

Name: ______________________________

Activity #1

1.) Compute the products below.

$-3 \times 5 =$
$-3 \times 4 =$
$-3 \times 3 =$
$-3 \times 2 =$
$-3 \times 1 =$
$-3 \times 0 =$

Think About It!

- How does the product change as the numbers multiplied by -3 get smaller?______________
- Predict -3×-1. Explain your prediction.______________
- Write the next four equations in the pattern.______________

2.) Compute the products below.

$-5 \times 5 =$
$-5 \times 4 =$
$-5 \times 3 =$
$-5 \times 2 =$
$-5 \times 1 =$
$-5 \times 0 =$

Think About It!

- How does the product change as the numbers multiplied by -5 get smaller?______________
- Predict -5×-1. Explain your prediction.______________
- Write the next four equations in the pattern.______________

3.) Compute the products below.

$-7 \times 5 =$
$-7 \times 4 =$
$-7 \times 3 =$
$-7 \times 2 =$
$-7 \times 1 =$
$-7 \times 0 =$

Think About It!

- How does the product change as the numbers multiplied by -7 get smaller?______________
- Predict -7×-1. Explain your prediction.______________
- Write the next four equations in the pattern.______________

Activity #2

For the following questions, use positive numbers for running speeds to the right and negative numbers for running speeds to the left. Use positive numbers for time in the future and negative numbers for time in the past.

-50 -40 -30 -20 -10 0 10 20 30 40 50

1.) Andrew passes the 0 point running to the left at 5 meters per second. Where was he 6 seconds earlier? Write an equation that expresses your answer and explain how you determined your answer.______________

-50 -40 -30 -20 -10 0 10 20 30 40 50

2.) Mark passes the 0 point running to the left at 4 meters per second. Where was he 7 seconds earlier? Write an equation that expresses your answer and explain how you determined your answer.____

____

-50 -40 -30 -20 -10 0 10 20 30 40 50

3.) Hank passes the 0 point running to the left at 2 meters per second. Where was he 9 seconds earlier? Write an equation that expresses your answer and explain how you determined your answer.____

____

Think About It!

- Based on these activities, what can you conclude about the product of two negative numbers?

____

Assessment Activity:

For the assessment activity, the students will perform a series of computations using either addition or multiplication. The purpose of this activity is to assess if the students understand that the sum of two negative numbers is negative and the multiplication of two negative numbers is positive.

Solution:

1.) $-2 \times -4 = 8$ 2.) $-6 \times -3 = 18$ 3.) $-5 + (-2) = -7$

4.) $-8 + (-1) = -9$ 5.) $-3 \times -4 = 12$ 6.) $-2 + (-7) = -9$

Activity Sheet

Assessment Activity:

Compute the following.

1.) $-2 \times -4 =$

2.) $-6 \times -3 =$

3.) $-5 + (-2) =$

4.) $-8 + (-1) =$

5.) $-3 \times -4 =$

6.) $-2 + (-7) =$

Error Description: Fractions

Error:

When adding and multiplying fractions, students do not know when it is necessary to find a common denominator.

Reason for Error:

Students who do not have a conceptual understanding of fractions may simply memorize the procedures for adding and multiplying fractions. Then, without a conceptual understanding it becomes very easy for students to incorrectly apply and intertwine the two procedures. For example, students may unnecessarily find a common denominator when multiplying fractions. As a result, it is important to include both the addition and multiplication of fractions in the activities that address this error. However, it is important to note that finding a common denominator when multiplying fractions will not produce an incorrect answer, it is just not necessary nor very efficient to find a common denominator when multiplying fractions.

Error Description: Adding Fractions

Activity #1 Description:

In the first activity, students will manipulate pie pieces in order to add the fractions in the three given story problems. The students will then be asked to consider how the fractions can be changed in order to create an easier addition problem where the pie pieces are the same size. The purpose of this activity is to help the students understand how a common denominator can help them easily add fractions.

Solution:

1.) $\frac{2}{6}+\frac{3}{6}=\frac{5}{6}$ 2.) $\frac{5}{8}+\frac{2}{4}=\frac{9}{8}$ 3.) $\frac{2}{3}+\frac{1}{4}=\frac{11}{12}$

Activity #2 Description:

In the second activity, the students will be asked to fill in the corresponding missing numbers on the given number line. Then, the students will use the number line to help them add fractions.

Solution:

Number Line:

Row 2: $\frac{1}{3}$ Row 3: $\frac{2}{4}$ Row 4: $\frac{1}{6},\frac{4}{6},\frac{5}{6}$

1.) $\frac{1}{2}+\frac{3}{4}=\frac{5}{4}$ 2.) $\frac{1}{3}+\frac{1}{2}=\frac{5}{6}$ 3.) $\frac{1}{3}+\frac{4}{6}=\frac{6}{6}=1$

Activity #3 Description:

In the third activity, the students will create a flowchart describing the steps that they use to add the given fraction. This activity allows students to demonstrate their understanding of adding fractions and helps them organize the procedure that they used so that the procedure can be easily recalled for future use.

Solution: (Students' answers may vary.)

$$\frac{1}{3}+\frac{1}{6}=\frac{1}{2}$$

Multiplied $\frac{1}{3}$ by 2 to get a common denominator.

$$\frac{1}{3}\times 2=\frac{2}{6}$$

$$\frac{2}{6}+\frac{1}{6}$$

Added the numerators and recorded the sum.

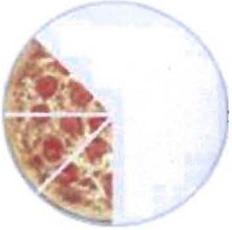
$$2+1=3$$

Drew a line under the sum and transferred over the same denominator.

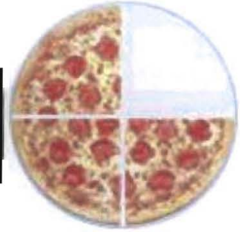
$$\frac{3}{6}$$

Divided both the numerator and the denominator by 2 to put the fraction into lowest terms.

$$\frac{1}{2}$$



Activity Sheet



Name: ______________________________________

Activity #1:

Complete the following problems.

1.) Jane says that she can eat $\frac{2}{6}$ of a circular pizza and Kari says she can eat $\frac{3}{6}$ of a pizza. Use pie pieces to determine how much pizza Jane and Kari can eat altogether.

Think About It!

- How much pizza can Jane and Kari eat altogether?______________________________

2.) Dave says he can eat $\frac{5}{8}$ of a circular pizza and Jon says he can eat $\frac{2}{4}$ of a pizza. Use pie pieces to determine how much pizza Dave and Jon can eat altogether.

Think About It!

- How much pizza can Dave and Jon eat altogether?______________________________
- How can we change the problem into one that is just like 1.) where the parts are the same?______________________________

3.) Once they ordered their pizzas, Dave actually ate $\frac{2}{3}$ of a pizza and Jon only ate $\frac{1}{4}$ of a pizza. Use the pie pieces to determine how much pizza Dave and Jon can eat altogether.

Think About It!

- How much pizza did Dave and Jon altogether?______________________________
- How can we change the problem into one that is just like 1.) where the parts are the same?______________________________

Activity #2:

Fill in the blanks with the corresponding fractions below. Then, use the number line to help you solve the addition problems below.

0 ____ 1

$\frac{1}{2}$

__ $\frac{2}{3}$

$\frac{1}{4}$ __ $\frac{3}{4}$

__ $\frac{2}{6}$ $\frac{3}{6}$ __ __

1.) $\frac{1}{2}+\frac{3}{4}=$

Think About It!

- How did you add the fractions?______________________________________
- Why?____

2.) $\frac{1}{3}+\frac{1}{2}=$

Think About It!

- How did you add the fractions?______________________________________
- Why?____

3.) $\frac{1}{3}+\frac{4}{6}=$

Think About It!

- How did you add the fractions?______________________________________
- Why?____

Activity #3:

Create a flowchart of the steps you use to add the following two fractions.

$$\frac{1}{3} + \frac{1}{6} = \_\_\_\_\_\_$$

Error Description: Multiplying Fractions

Activity #1 Description:

In the first activity, students will review two examples of how a symbolic expression involving multiplication can be represented using words. Then, the students must convert the given symbolic expressions into words. Next, the students will use the phrases they constructed to help them determine the product of a few of the symbolic expressions. The purpose of this activity is for the students to realize what the multiplication of two fractions represents conceptually.

Solution:

1.) 3×5	3 groups of 5	15 total
2.) 7×2	7 groups of 2	14 total
3.) $\frac{1}{2} \times 8$	$\frac{1}{2}$ of 8	4 total
4.) $2 \times \frac{1}{4}$	2 groups of $\frac{1}{4}$	$\frac{2}{4}$ or $\frac{1}{2}$ total

1.) $\frac{1}{3} \times \frac{3}{4}$	$\frac{1}{3}$ of $\frac{3}{4}$
2.) $\frac{2}{3} \times \frac{5}{6}$	$\frac{2}{3}$ of $\frac{5}{6}$
3.) $\frac{1}{2} \times \frac{1}{2}$	$\frac{1}{2}$ of $\frac{1}{2}$

- Based on your written statement, determine the product of $\frac{1}{2} \times \frac{1}{2}$. $\frac{1}{4}$

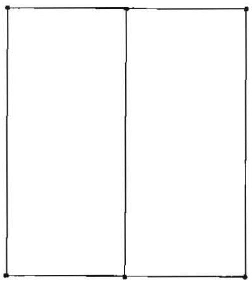
Activity #2 Description:

In the second activity, students will fold a piece of paper to illustrate the multiplication of two fractions. The purpose of this activity is to help the students understand the multiplication of fractions conceptually using a visual representation.

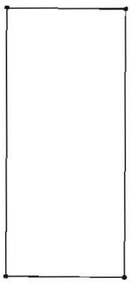
Solution:

1.) Fold the square of paper in front of you in half. Then, fold it back so that only $\frac{1}{2}$ is showing. Now, turn the paper 90 degrees. Then, fold the paper into fourths. Lastly, fold the paper back so that only $\frac{3}{4}$ is showing. Count the number of squares that remain on the paper. Record this number in the numerator. Lastly, unfold the paper completely and count the total number of squares. Record this number in the denominator.

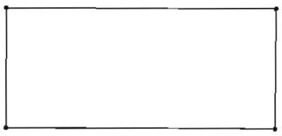
Step 1:



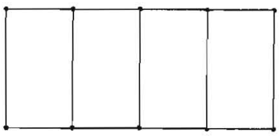
Step 2:



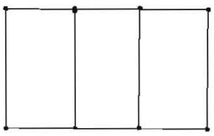
Step 3:



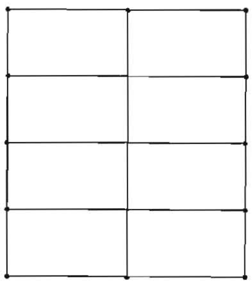
Step 4:



Step 5: $\frac{3}{?}$

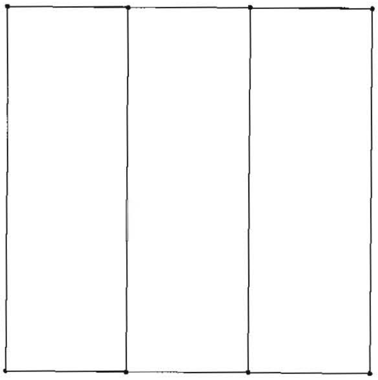


Step 6: $\frac{3}{8}$

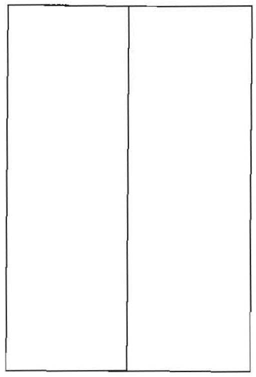


2.) Fold the square of paper in front of you into thirds. Then, fold it back so that only $\frac{2}{3}$ is showing. Now, turn the paper 90 degrees. Then, fold the paper into fourths. Lastly, fold the paper back so that only $\frac{3}{4}$ is showing. Count the number of squares that remain on the paper. Record this number in the numerator. Lastly, unfold the paper completely and count the total number of squares. Record this number in the denominator.

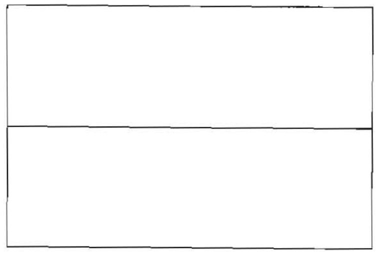
Step 1:



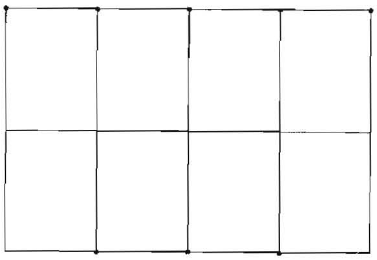
Step 2:



Step 3:

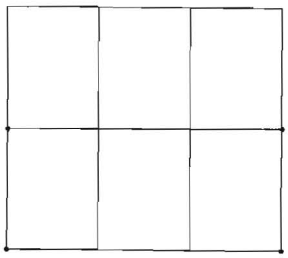


Step 4:

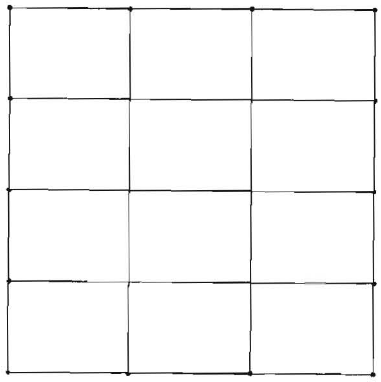


Step 5:

$\frac{6}{?}$

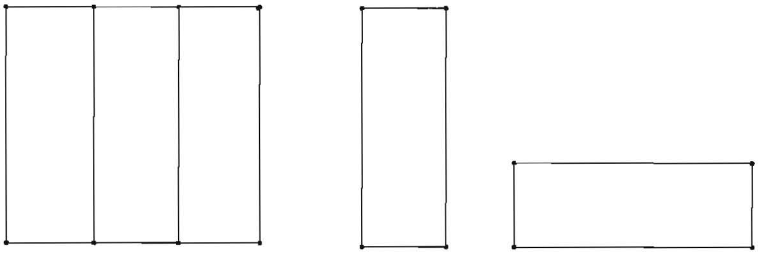


Step 6: $\frac{6}{12}$

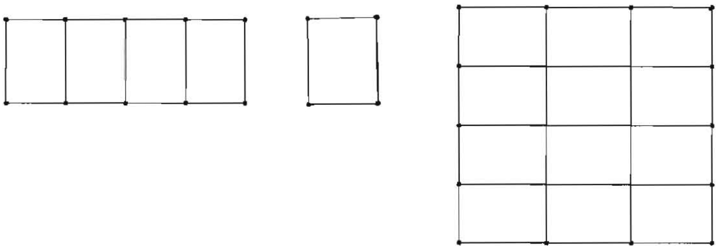


3.) Fold the square of paper in front of you into thirds. Then, fold it back so that only $\frac{1}{3}$ is showing. Now, turn the paper 90 degrees. Then, fold the paper into fourths. Lastly, fold the paper back so that only $\frac{1}{4}$ is showing. Count the number of squares that remain on the paper. Record this number in the numerator. Lastly, unfold the paper completely and count the total number of squares. Record this number in the denominator.

Step 1: **Step 2:** **Step 3:**



Step 4: **Step 5:** $\frac{1}{?}$ **Step 6:** $\frac{1}{12}$



Activity #3 Description:

In order to self-assess their understanding of multiplying fractions, the students will create a flowchart of the process they follow when multiplying two given fractions.

Solution: (Students' answers may vary.):

$$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15}$$

Multiply the numerators and record the product. $2 \times 3 = 6$	→	Multiply the denominators. $3 \times 5 = 15$	→	Record the product of the denominators beneath the product of the numerators. $\frac{6}{15}$

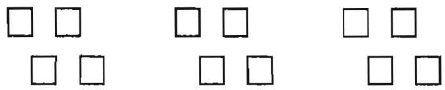
Activity Sheet

Name: ______________________________

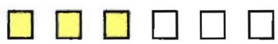
Activity #1:

Review the following examples and complete the questions.

Example #1
Jon built **3 groups of 4** with the blocks. When asked how many blocks he had total, Jon found the total by multiplying 3×4. Jon determined that he had 12 blocks total.



Example #2
Dave put six blocks in a row. Then, Dave was asked how many of the blocks are yellow. Dave stated that **½ of 6** blocks are yellow. Dave found the number of yellow blocks by multiplying $\frac{1}{2} \times 6$. Dave determined that he had 3 yellow blocks.



Express the following symbolic expressions using words. Then, determine the product of each expression.

1.) 3×5 ______________________________

2.) 7×2 ______________________________

3.) $\frac{1}{2} \times 8$ ______________________________

4.) $2 \times \frac{1}{4}$ ______________________________

Now, based on the examples above, express the following symbolic expressions using words.

1.) $\frac{1}{3} \times \frac{3}{4}$ ____________________

2.) $\frac{2}{3} \times \frac{5}{6}$ ____________________

3.) $\frac{1}{2} \times \frac{1}{2}$ ____________________

Think About It!

- Based on your written statement, determine the product of $\frac{1}{2} \times \frac{1}{2}$. ____________
- How did you determine your answer? ____________________

____

Activity #2:

1.) Jane has $\frac{1}{2}$ of a square pizza left. If she gives $\frac{3}{4}$ of the leftover pizza to Kari, how much of the whole pizza will Kari receive.

Let the square piece of paper attached to this activity sheet represent the pizza. Use the piece of paper to determine how much of the whole pizza Kari will get.

Think About It!

- How much of the whole pizza did Kari get? ____________________
- How does this amount relate to the fractions in the story problem? ____________________

____

2.) Dave has $\frac{2}{3}$ of a square pizza left. If he gives $\frac{3}{4}$ of the leftover pizza to Jon, how much of the whole pizza will Jon receive.

Let the square piece of paper attached to this activity sheet represent the pizza. Use the piece of paper to determine how much of the whole pizza Jon will get.

Think About It!

- How much of the whole pizza did Jon get? ____________________
- How does this amount relate to the fractions in the story problem? ____________________

____

2.) Matt has $\frac{1}{3}$ of a square pizza left. If he gives $\frac{1}{4}$ of the leftover pizza to James, how much of the whole pizza will James receive.

Let the square piece of paper attached to this activity sheet represent the pizza. Use the piece of paper to determine how much of the whole pizza James will get.

Think About It!

- How much of the whole pizza did James get?____________________________________
- How does this amount relate to the fractions in the story problem?____________________

____

- Based on the relationship you found in the previous question, develop a procedure for multiplying fractions.____

____

Activity #3:

Create a flowchart of the steps you use to multiply the following two fractions.

$\frac{2}{3} \times \frac{3}{5} =$ ______

Assessment Activity:

The purpose of this activity is to ensure that the students understand how to multiply and add fractions. In general, it is important that the students can recognize the differences in the procedures for adding and multiplying fractions.

Solution:

1.) $\frac{5}{6} \times \frac{7}{8} = \frac{35}{48}$

2.) $\frac{3}{4} + \frac{7}{8} = \frac{6}{8} + \frac{7}{8} = \frac{13}{8}$

3.) $\frac{1}{9} + \frac{4}{5} = \frac{5}{45} + \frac{36}{45} = \frac{41}{45}$

4.) $\frac{3}{5} \times \frac{2}{7} = \frac{6}{35}$

5.) $\frac{3}{8} + \frac{1}{3} = \frac{9}{24} + \frac{8}{24} = \frac{17}{24}$

6.) $\frac{4}{7} \times \frac{3}{4} = \frac{12}{28} = \frac{3}{7}$

Activity Sheet

Assessment Activity:

Compute the following.

1.) $\frac{5}{6} \times \frac{7}{8} =$

2.) $\frac{3}{4} + \frac{7}{8} =$

3.) $\frac{1}{9} + \frac{4}{5} =$

4.) $\frac{3}{5} \times \frac{2}{7} =$

5.) $\frac{3}{8} + \frac{1}{3} =$

6.) $\frac{4}{7} \times \frac{3}{4} =$

Error Description: Decimals

Error:

Students believe that they must line up the decimal points in order to multiply decimals.

Reason for Error:

This error is a result of overspecializing. Students have learned that they must line up the decimal points when adding and subtracting decimals and students falsely apply this method to the multiplication of decimals. Therefore, the students are placing inappropriate restrictions on the multiplication of decimals. However, it is important to note that lining up the decimals does not result in a wrong answer. Lining up the decimals is simply unnecessary and is not the most efficient way to multiply decimals.

Activity #1 Description:

In the first activity, the students will compare the product of two decimals with the product of two whole numbers in which the digits are same but there are no decimal points. The students will then be asked to determine how the products are similar and how the products are different. The purpose of this activity is to help the students realize that the digits in the products are the same. The only difference in the products is the decimal point. The students will also be asked to determine the relationship between the placement of the decimal points in the factors and the placement of the decimal point in the product.

Solution:

1.) $24 \times 63 = 1{,}512$	1.) $234 \times 64 = 14{,}976$	1.) $456 \times 14 = 6{,}384$
2.) $2.4 \times 6.3 = 15.12$	2.) $2.34 \times 6.4 = 14.976$	2.) $4.56 \times .14 = .6384$

Activity #2 Description:

In the second activity, the students will compute the product of two whole numbers. Then, using only this computation, estimation, and the relationship they discovered in Activity #1 about the placement of the decimal point in the product, the students will determine the product of several pairs of decimals that consist of the same digits. This activity is an expansion of Activity #1. The purpose of this activity is for students to apply their understanding of the relationship between the number of decimal points in the factors and the placement of the decimal in the product.

Solution:

1.) $12 \times 17 = 204$

1.) $1.2 \times 1.7 = 2.04$
2.) $12 \times 1.7 = 20.4$
3.) $0.12 \times 0.17 = 0.0204$
4.) $1.2 \times 0.17 = 0.204$

2.) $32 \times 43 = 1{,}376$

1.) $0.32 \times 4.3 = 1.376$
2.) $32 \times 0.43 = 13.76$
3.) $3.2 \times 43 = 137.6$
4.) $0.32 \times 0.43 = 0.1376$

3.) $151 \times 13 = 1{,}963$

1.) $1.51 \times 1.3 = 1.963$
2.) $151 \times 0.13 = 19.63$
3.) $15.1 \times 13 = 196.3$
4.) $1.51 \times 0.13 = 0.1963$